

An Adaptive Windowed Approach for Frequency Estimation in Power Systems

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Abstract: A novel technique for online estimation of the fundamental frequency of power system in both the single-phase and three-phase cases is proposed. This is achieved based on the consideration of the relationship between the samples within every three consecutive sliding windows, and the use of an adaptive filter trained by the least mean square (LMS) algorithm. The evolution of the adaptive weight coefficient within the adaptive filter is used to estimate the system frequency in a recursive and robust manner. Compared with the original work which employs the Wiener filtering approach, the proposed method alleviates the drawbacks of Wiener filtering, such as sensitivity to noise and harmonics, at a cost of convergence time. Simulations in both benchmark synthetic cases and for real-world scenarios support the analysis.

1. Introduction

Accurate frequency estimation in the power system is a key power quality parameter. Its importance has become even more emphasized owing to the trends for electricity industry deregulation [1], and the subsequent penetration of distributed generation into the power grid [2], [3], where the frequency variations occur as a consequence of the dynamic unbalance between the generation and the load. A system frequency that is lower than the nominal value indicates an overloaded power system, while a frequency higher than the nominal value implies more generation than the connected load [4].

To deal with these issues in a timely and efficient way, fast and accurate frequency estimation has attracted much attention. A variety of architectures and algorithms have been developed for this purpose, including zero crossing techniques [1], [5], discrete Fourier transform (DFT) based algorithms [6], [7], phase-locked loops (PLL) [8], [9], least-squares based adaptive filters [4], [10]–[12], and Kalman filters [13], [14]. The lack of robustness to noise and harmonics associated with standard techniques can be attributed to both “single point estimates” in many current algorithms, sensitivity due to the feedback loop in PLL, and the inability to produce correct estimates over short data segments and during transients of DFT based methods. In addition, very few current methods are well equipped to deal with unbalanced system conditions.

Of particular interest to this work are techniques that benefit from the mathematical advantages provided by employing a relation between the information available in consecutive data samples [15]–[17]. Recently, a new algorithm that employs a sliding window over several successive samples has been proposed in [18], and has been shown to exhibit enhanced robustness as compared with the signal point frequency estimators. Based on a neat mathematical modeling of the relation between three successive data windows, the instantaneous frequency is extracted using the Wiener filter. Compared with traditional methods based on joint modeling of consecutive samples [16], this algorithm can be considered a generic extension from a scalar into a vector windowed form, where more available observations are involved in the calculation, enable this method to outperform its scalar counterpart. However, the main drawback, inherited from the limitations of Wiener filtering, remains its sensitivity to noise and harmonics.

To this end, we here present an adaptive windowed algo-rithm based on the least-mean-square (LMS) adaptive filter, in order to recursively estimate the mathematical relation between the three consecutive moving windows of data samples, whereby the instantaneous frequency is estimated from the associated weight coefficient. This provides additional flexibility in the estimation, robustness to harmonics and noise, and the ability to operate in unbalanced system conditions, where the Wiener filter based approaches fail. The proposed method also offers computational advantages, as Wiener filters are based on a matrix inversion of potentially ill-conditioned correlation matrices, causing serious stability issues.

2. An optimal windowed approach

Consider a discrete time voltage signal, given by

$$v(k) = A \cos(\omega k \Delta T + \phi) \quad (1)$$

where A is the peak value of the voltage $v(k)$, k time instant, $\Delta T = 1/f_s$ sampling interval, f_s sampling frequency, ϕ phase of fundamental component, and $\omega = 2\pi f$ the angular frequency, with f being the system frequency to be estimated.

In a similar way, the previous and following time instants, $v(k-1)$ and $v(k+1)$, are represented as

$$v(k-1) = A \cos(\omega(k-1)\Delta T + \phi) \quad (2)$$

$$v(k+1) = A \cos(\omega(k+1)\Delta T + \phi) \quad (3)$$

and can be further expanded into

$$\begin{aligned} v(k-1) &= A \cos(\omega k \Delta T + \phi - \omega \Delta T) \\ &= A \cos(\omega k \Delta T + \phi) \cos(\omega \Delta T) + A \sin(\omega k \Delta T + \phi) \sin(\omega \Delta T) \end{aligned} \quad (4)$$

and

$$\begin{aligned} v(k+1) &= A \cos(\omega k \Delta T + \phi + \omega \Delta T) \\ &= A \cos(\omega k \Delta T + \phi) \cos(\omega \Delta T) - A \sin(\omega k \Delta T + \phi) \sin(\omega \Delta T) \end{aligned} \quad (5)$$

A relationship connecting the three consecutive voltage sam-ples can be obtained by adding (4) and (5) to give [18]

$$\begin{aligned} v(k-1) + v(k+1) &= 2A \cos(\omega k \Delta T + \phi) \cos(\omega \Delta T) \\ &= 2v(k) \cos(\omega \Delta T) \end{aligned} \quad (6)$$

The instantaneous frequency can now be estimated as

$$\hat{f}(k) = \frac{f_s}{2\pi} \cos^{-1} \left(\frac{v(k-1) + v(k+1)}{2v(k)} \right) \quad (7)$$

Despite the elegant and closed form, a direct usage of (7) causes several problems. First, the frequency estimate $\hat{f}(k)$ depends on only three samples, making it very sensitive to noise and harmonics, and a filtering pre-conditioning scheme is necessary as a preprocessing step when the voltage signal is contaminated by harmonics. Second, large estimation errors may be obtained when the sample $v(k)$ is in a region near the zero crossing, a case even more emphasized when the sampling frequency f_s is high.

To help to deal with these deficiencies, a moving window scheme, proposed in [18], recognizes that the relationship among the scalars $\{v(k-1), v(k), v(k+1)\}$ in (6) holds at each time instant k , and the algorithm considers three windows of multiple samples with length $(2L+1)$, where

$\{v(k-1), v(k), v(k+1)\}$ are at the window center, so that

$$\begin{aligned}\mathbf{v}(k-1) &\triangleq [v(k-L-1), \dots, v(k+L-1)]^T \\ \mathbf{v}(k) &\triangleq [v(k-L), \dots, v(k+L)]^T \\ \mathbf{v}(k+1) &\triangleq [v(k-L+1), \dots, v(k+L+1)]^T\end{aligned}\quad (8)$$

This way, the following vector-valued version of (6) holds

$$\frac{\mathbf{v}(k-1) + \mathbf{v}(k+1)}{2} = \mathbf{v}(k) \cos(\omega \Delta T) \quad (9)$$

In [18], a Wiener filtering approach [19] was employed to obtain a least-squares estimate of $\cos(\omega \Delta T)$, that is

$$\cos(\hat{\omega} \Delta T) = \frac{\mathbf{v}^T(k) (\mathbf{v}(k-1) + \mathbf{v}(k+1))}{2 \mathbf{v}^T(k) \mathbf{v}(k)} \quad (10)$$

from which the system frequency is estimated as

$$\hat{f}(k) = \frac{f_s}{2\pi} \cos^{-1} \left(\frac{\mathbf{v}^T(k) \mathbf{v}(k-1) + \mathbf{v}^T(k) \mathbf{v}(k+1)}{2 \mathbf{v}^T(k) \mathbf{v}(k)} \right) \quad (11)$$

Observe that an accurate computation of the inverse cosine function in (11) has disadvantages in practical implementation of the algorithm, from a time-consuming calculation to enhanced sensitivity in certain regions of the voltage waveform due to $v(k)$ being in the denominator of (7). One way to overcome this shortcoming is to use the Taylor series expansion of (9), resulting in a reduced estimation accuracy but a much more mathematically tractable expression. In this light, consider the frequency ω in terms of an instantaneous change $\Delta\omega$ around its nominal value ω_0 ; then (9) can be expanded as

$$\begin{aligned}\frac{\mathbf{v}(k-1) + \mathbf{v}(k+1)}{2} &= \mathbf{v}(k) \cos(\omega_0 \Delta T) \cos(\Delta\omega \Delta T) \\ &\quad - \mathbf{v}(k) \sin(\omega_0 \Delta T) \sin(\Delta\omega \Delta T)\end{aligned}\quad (12)$$

Since in practical applications the frequency ω does not deviate significantly from its nominal value $\omega_0 = 2\pi 50\text{Hz}$, and in standard systems, $\Delta T = 1/f_s$ is very small, the corresponding first order Taylor series approximations gives $\cos(\Delta\omega \Delta T) \approx 1$ and $\sin(\Delta\omega \Delta T) \approx \Delta\omega \Delta T$, so that

$$\frac{\mathbf{v}(k-1) + \mathbf{v}(k+1)}{2} \approx \mathbf{v}(k) [\cos(\omega_0 \Delta T) - \Delta\omega \Delta T \sin(\omega_0 \Delta T)] \quad (13)$$

Given that $\omega_0 = 2\pi f_0$, $\Delta\omega = 2\pi \Delta f$ and $f_s = N f_0$, this allows us to write

$$a [b \mathbf{v}(k) - \mathbf{v}(k-1) - \mathbf{v}(k+1)] = \mathbf{v}(k) \Delta f \quad (14)$$

where $a = f_s / (4\pi \sin(2\pi / N))$, and $b = 2 \cos(2\pi / N)$. A Wiener filtering solution to this equation results in the estimate of the deviation Δf , as given in [18]

$$\Delta \hat{f}(k) = a \mathbf{v}^T(k) \frac{b \mathbf{v}(k) - \mathbf{v}(k-1) - \mathbf{v}(k+1)}{\mathbf{v}^T(k) \mathbf{v}(k)} \quad (15)$$

allowing us to arrive at the system frequency estimate in the form

$$\hat{f}(k) = f_0 + \Delta \hat{f}(k) \quad (16)$$

3. The proposed adaptive windowed approach

Wiener filtering is a back approach which involves the calculation of matrix inverse, and is thus sensitive to noise and harmonics. In addition, in its block calculation, the filter weights are kept fixed, making such an approach ill equipped with the need to operate in unbalanced system conditions and in the presence of transients and random disturbances. To deal with these issues, we here employ the least mean square (LMS) algorithm, widely used in adaptive signal processing applications. We show that the evolution of the corresponding weight coefficient can be used to estimate the system frequency recursively and in real time, while at the same time giving unbiased estimates. In the context of adaptive filtering, the right hand side of (14) can be estimated by a linear model, where

$$\mathbf{y}(k) = \mathbf{v}(k)w(k) \quad (17)$$

where $\mathbf{y}(k)$ is the filter output, and $w(k)$ is the filter weight (coefficient) at time instant k . The estimation error $\mathbf{e}(k)$ and the cost function $\mathcal{J}(k)$ can be respectively defined as

$$\mathbf{e}(k) = a(b\mathbf{v}(k) - \mathbf{v}(k-1) - \mathbf{v}(k+1)) - \mathbf{y}(k) \quad (18)$$

$$\mathcal{J}(k) = \frac{1}{2} \mathbf{e}^T(k) \mathbf{e}(k) \quad (19)$$

where $a(b\mathbf{v}(k) - \mathbf{v}(k-1) - \mathbf{v}(k+1))$ serves as the desired signal that we wish to estimate. The weight update of the adaptive coefficient $w(k)$ can be obtained using a steepest descent approach as

$$w(k+1) = w(k) - \mu \nabla_w \mathcal{J}(k) \quad (20)$$

where μ is the step-size that controls the trade-off between the convergence speed and steady-state estimation error variance [19]. The gradient in (20) can be derived in the form

$$\begin{aligned} \mathcal{J}(k) &= \frac{1}{2} \mathbf{e}^T(k) \frac{\partial \mathbf{e}(k)}{\partial \mathbf{w}(k)} + \left(\frac{\partial \mathbf{e}(k)}{\partial \mathbf{w}(k)} \right)^T \mathbf{e}(k) \\ &= -\mathbf{e}^T(k) \frac{\partial \mathbf{y}(k)}{\partial w(k)} = -\mathbf{e}^T(k) \mathbf{v}(k) \end{aligned} \quad (21)$$

Hence, the recursive weight update takes the form

$$w(k+1) = w(k) + \mu \mathbf{e}^T(k) \mathbf{v}(k) \quad (22)$$

and the estimated transient frequency and the instantaneous system frequency are calculated as

$$\Delta \hat{f}(k) = w(k) \quad (23)$$

$$\hat{f}(k) = f_0 + \Delta \hat{f}(k) \quad (24)$$

The stability of such a closed-loop adaptive system based on the windowed-LMS is addressed in Appendix A. Note that the proposed windowed LMS scheme is not limited to the voltage modeling based on (6), it can be applied to extend any frequency estimation method which uses relationships between consecutive voltage samples based on a pure sinusoidal signal model, such as those in [4], [11], [20], [21].

A Three-Phase Windowed LMS Approach. In a three-phase power system, if line-to-line voltages are considered, no single-phase frequency estimation method adequately characterizes system frequency, because up to six different single-phase voltage signals may exist [22]. Therefore, an optimal solution would be based on a framework which simultaneously considers all the three-phase voltages, thus enabling a unified estimation of system frequency as a whole, and providing enhanced robustness. To this end, we extend the proposed Windowed LMS approach to

cater for the system frequency estimation in a three-phase power system. Consider the following three-phase power system

$$\begin{aligned} v_a(k) &= V_a(k) \cos(\omega k \Delta T + \phi) \\ v_b(k) &= V_b(k) \cos\left(\omega k \Delta T + \phi - \frac{2\pi}{3}\right) \\ v_c(k) &= V_c(k) \cos\left(\omega k \Delta T + \phi + \frac{2\pi}{3}\right) \end{aligned} \quad (25)$$

where $V_a(k), V_b(k), V_c(k)$ are the peak values of each fundamental voltage component, and the remaining parameters are as defined in Section II. We now introduce a composite sliding window which comprises $2L+1$ consecutive samples in all the three phases as:

$$\mathbf{v}_{3p}(k) = [\mathbf{v}_a^T(k), \mathbf{v}_b^T(k), \mathbf{v}_c^T(k)]^T \quad (26)$$

It therefore still holds that

$$\frac{\mathbf{v}_{3p}(k-1) + \mathbf{v}_{3p}(k+1)}{2} = \mathbf{v}_{3p}(k) \cos(\omega \Delta T) \quad (27)$$

Similarly to the analysis in Section II, using the first order Taylor series approximation, we arrive at

$$a(b\mathbf{v}_{3p}(k) - \mathbf{v}_{3p}(k-1) - \mathbf{v}_{3p}(k+1)) = \mathbf{v}_{3p}(k) \Delta f_{3p}$$

and the corresponding windowed LMS algorithm for the estimation of the system frequency can be summarized as

$$\begin{aligned} \mathbf{y}(k) &= \mathbf{v}_{3p}(k) w_{3p}(k) \\ \mathbf{e}(k) &= a(b\mathbf{v}_{3p}(k) - \mathbf{v}_{3p}(k-1) - \mathbf{v}_{3p}(k+1)) - \mathbf{y}(k) \\ \Delta \hat{f}_{3p} &= w_{3p}(k) \\ \hat{f}_{3p}(k) &= f_0 + \Delta \hat{f}_{3p} \\ w_{3p}(k+1) &= w_{3p}(k) + \mu_{3p} \mathbf{e}^T(k) \mathbf{v}_{3p}(k) \end{aligned} \quad (28)$$

Note that the filter length of the windowed LMS in the three-phase case is three times that of the windowed LMS in a single phase case. Following the stability analysis in Appendix A, when the three-phase power system is in its normal operation, that is, $\sigma_{v_a}^2 = \sigma_{v_b}^2 = \sigma_{v_c}^2 = \sigma_v^2$, the range of $\mu_{3\text{-phase}}$ that guarantees the stability is given by

$$0 < \mu_{3p} < \frac{2}{3(2L+1)\sigma_v^2} \quad (29)$$

4. Simulations

To illustrate the benefits of the proposed windowed LMS method in both the single-phase and three-phase cases, compared with the Winer filtering solution, simulations for several typical power system operating conditions were conducted in the Matlab programming environment. The step-size of a one-phase LMS was set to $\mu = 0.02$. Owing to the fact that the effective filter length of a three-phase LMS is three times that of the one-phase LMS, to make a fair comparison, the step-size of three-phase LMS was set to a third of μ . The performances of all the algorithms were quantified by the mean square error (MSE) in dB, defined as

$$\text{MSE(dB)} = 10 \log_{10} \left(\sum_{k=1}^K (f_0 - \hat{f}(k))^2 / K \right) \quad (30)$$

where the MSE is evaluated over 0.5 sec. The simulated power system was in its normal

operating condition at 50 Hz, with a balanced three-phase voltages with unity magnitudes.

4.1 Synthetic Benchmark Cases.

In the first set of simulations, we considered the performance in a noisy environment with the signal to noise ratio (SNR) = 60 dB. Fig. 1 illustrates the estimation performances, with the three-phase voltages sampled at $f_s = 1000\text{Hz}$ and the window size of $2L+1=3$. Due to the adaptive nature of LMS algorithms, both needed around 0.15 sec to converge, whereas the Wiener filtering solutions (being calculated backwards) obtained instantaneously frequency estimates but were not able to provide optimal estimates in the noisy environment as illustrated by a large variability of the estimate throughout the time segment considered. The robustness of LMS approaches against noise can be observed at the steady state, after convergence, e.g. after 0.15 sec. Fig. 2 illustrates the estimated MSEs of all the algorithms against different levels of white Gaussian noise. The windowed LMS solutions exhibited improved performance over the Wiener based solutions by around 40 dB and 30 dB, respectively, for high SNR and smaller but significant improvement for low SNR. The 3-phase windowed LMS maintained around 5 dB improvement as compared with its 1-phase counterpart in the high SNR region, however, this advantage reduced when the SNR was below 40 dB.

Fig. 3 illustrates the impact of the sampling frequency f_s on the estimation performance of all the algorithms investigated. Observe that, when f_s increases, MSEs grows monotonically in both noise-free and noisy cases, however, the advantage of windowed LMS over Wiener filtering was always maintained.

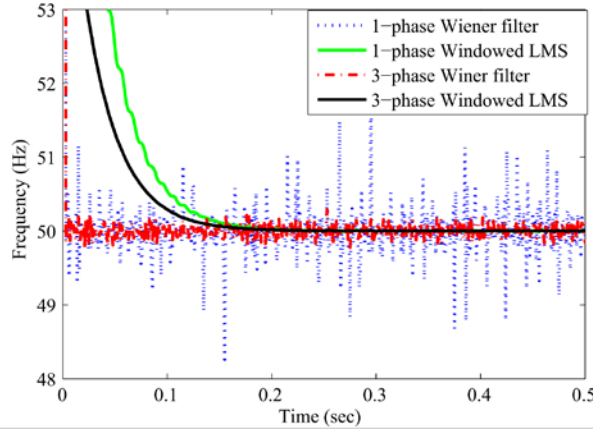


Fig. 1. Frequency estimation in noisy conditions with SNR=60 dB.

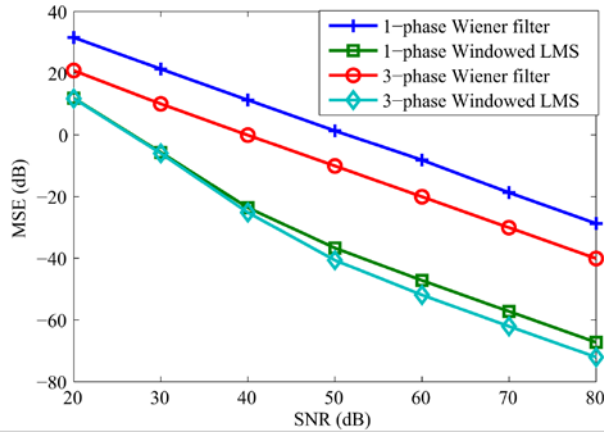


Fig. 2. Frequency estimation performance of all the algorithms under different noise levels, obtained by averaging 500 independent trials.

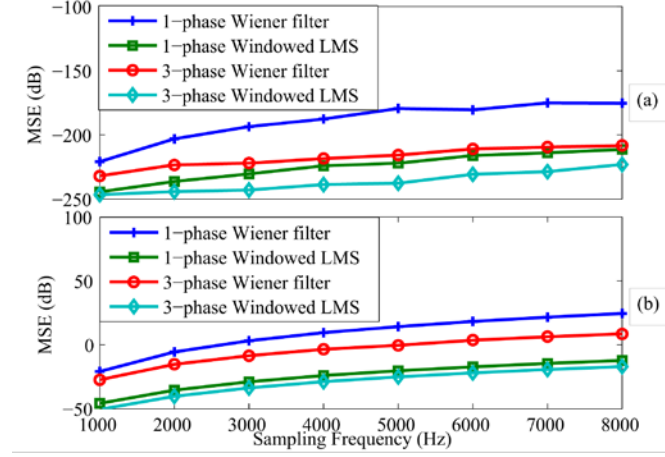


Fig. 3. Effect of the sampling frequency f_s on the MSEs of all algorithms, for a data window size of $2L+1 = 7$. (a) noise-free case (b) SNR=60 dB.

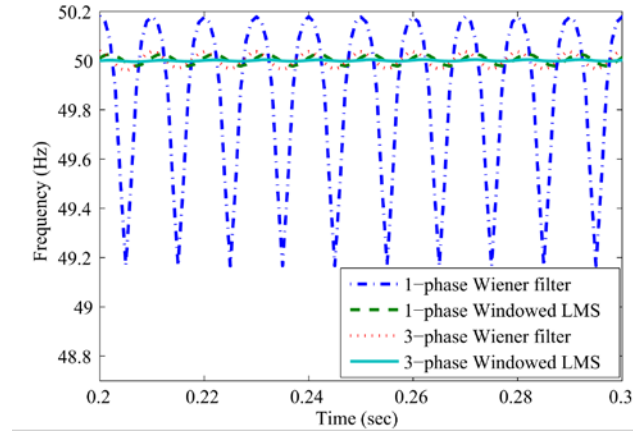


Fig. 4. Frequency estimation of all the algorithms with a data window size of $2L+1 = 5$ on the distorted power system with harmonics

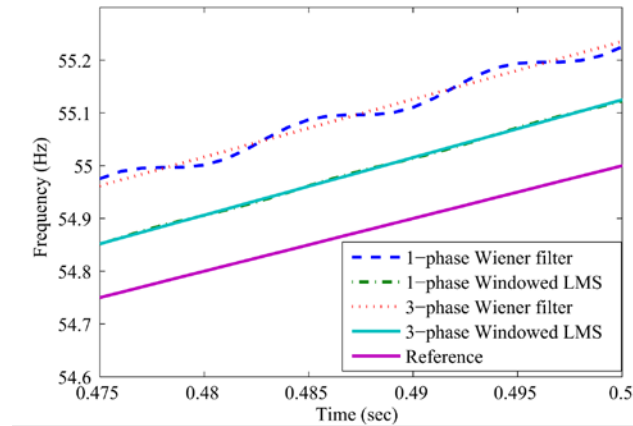
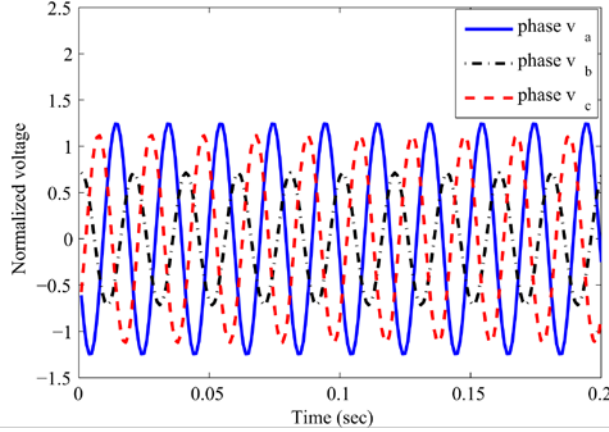


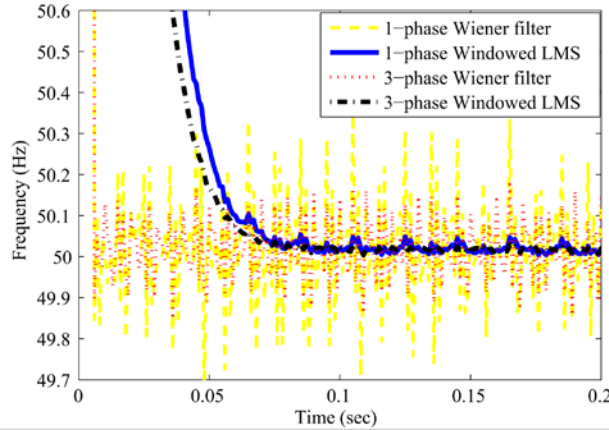
Fig. 5. Frequency estimation performance of all the algorithms for the case of the system frequency rise at a rate of 5 Hz/sec.

In the next set of simulations, we addressed the sensitivity of the frequency estimation algorithms to higher order harmonics. When the voltage is contaminated with harmonics, the estimated frequency is subject to an unavoidable oscillatory steady-state error. This is because the relationship between three consecutive voltage samples, expressed in (6), does not hold when the harmonics are also involved, especially affecting those frequency estimation algorithms for which the relationship between samples is based on a pure sinusoidal signal model [4]. To mitigate this problem, signal prefiltering is usually performed. The distorted power system considered here was

contaminated by a 20% third and fifth harmonic, respectively, and a 6th order bandpass FIR filter with cutoff frequencies of 20 Hz and 90 Hz was used to preprocess the voltages. Fig. 4 illustrates the steady-state performance of all the algorithms in this scenario. Even when the prefiltering was performed, harmonic distortion deteriorated the performance of the 1-phase Wiener filtering approach. On the other hand, both the 1-phase windowed LMS and 3-phase Wiener filtering approach obtained acceptable performance, with the MSEs of



(a) Time series of the real-world balanced three-phase voltages.



(b) Frequency estimation of all the algorithms for a data window size of $2L+1=9$.

Fig. 6. Frequency estimation for a real-world balanced three-phase power system.

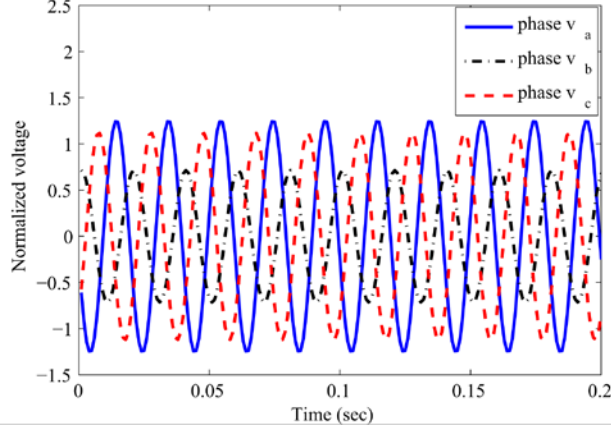
-35.2083 dB and -31.9184 dB respectively. The 3-phase windowed LMS achieved the best estimation performance at a -52.5127 dB MSE conforming with the analysis, and the estimated frequency was within the range between 49.998 Hz and 50.004 Hz.

Fig. 5 shows the frequency tracking abilities of all the algorithms studied, where the 50 Hz fundamental frequency of the three-phase power system underwent a rise at a rate of 5 Hz/s. A portion of the estimation results between 0.475 sec and 0.5 sec shows the windowed LMS methods approaching the reference frequency more closely as compared with Wiener filtering methods.

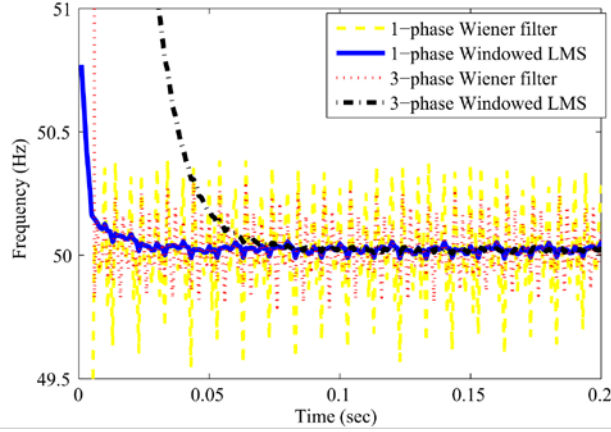
4.2 Real World Case Studies.

In the last set of simulations, real-world measurements were considered. The three-phase voltages were recorded at 110/20/10 kV transformer stations. The REL 531 numerical line distant protection terminal, produced by ABB Ltd., was installed in the station and was used to monitor changes in the three “phase-ground” voltages. The measured three “phase-ground” voltages with a system frequency of 50 Hz were sampled at 1 kHz and were normalized with respect to their normal peak voltage values. The first investigated power system was in a normal operation, and the balanced time-series of the three-phase voltages are shown in Fig. 6(a), the estimation results of the discussed algorithms with a data window size $2L+1=9$ is shown in Fig. 6(b). Both the 1-phase and 3-phase windowed LMS approaches achieved higher estimation accuracy in the steady state, as

compared with their Wiener filter counterparts, at an acceptable cost of 0.06 sec needed to converge, and the advantage of simultaneous consideration of three-phase voltages over single phase approaches can be observed through low variability of frequency estimates. The second investigated power system was in an unbalanced operation, where the phase voltage v_b experienced a short circuit with earth, causing the voltage to drop to 71.38% of its normal value, and at the same time, 24.38% and 11.85% voltage swells on v_a and v_c respectively, as illustrated in Fig. 7(a). The robustness of the proposed windowed approaches against the standard ones in unbalanced conditions can be observed in Fig. 7(b).



(a) Time-series of the real-world unbalanced three-phase voltages.



(b) Frequency estimation of all the algorithms for a data window size of $2L+1=9$.

Fig. 7. Frequency estimation for a real-world unbalanced three-phase power system

5. Conclusions

We have introduced an adaptive windowed least mean square (LMS) method for frequency estimation in both the 1-phase and 3-phase power systems. The evolution of the filter weight coefficient within the windowed LMS is used to estimate the system frequency in a recursive and real-time manner. The advantages of the proposed methods over the original Wiener filtering approaches have been illuminated over a range of power system conditions, such as in the presence of noise, harmonic distortion, frequency variation, and for real-world measurements on both balanced and unbalanced power systems.

Appendix A

This appendix outlines the theoretical stability analysis of the proposed windowed LMS frequency estimator. For this purpose, we consider the linear estimate of the desired signal via an adaptive filter in the form

$$a(b\mathbf{v}(k) - \mathbf{v}(k-1) - \mathbf{v}(k+1)) = \mathbf{v}(k)w_o \quad (31)$$

where w_o is the optimal weight coefficient. Following the analysis in [19], [23], we define the weight error coefficient as $\tilde{w}(k) = w_o - w(k)$. The evolution of the weight error coefficient can be analyzed based on (20) as

$$\tilde{w}(k+1) = \tilde{w}(k) - \mu \mathbf{e}^T(k) \mathbf{v}(k) \quad (32)$$

where the filter output error

$$\begin{aligned} \mathbf{e}(k) &= a(b\mathbf{v}(k) - \mathbf{v}(k-1) - \mathbf{v}(k+1)) - \mathbf{y}(k) \\ &= \mathbf{v}(k)(w_o - w(k)) = \mathbf{v}(k)\tilde{w}(k) \end{aligned} \quad (33)$$

Substituting (33) into (32) gives

$$\tilde{w}(k+1) = (1 - \mu \mathbf{v}^T(k) \mathbf{v}(k)) \tilde{w}(k) \quad (34)$$

Upon taking the statistical expectation, to achieve the stability of the proposed algorithm in the sense of convergence in the mean, we need to ensure that $|\tilde{w}(k+1)| < |\tilde{w}(k)|$, which gives

$$|1 - \mu E[\mathbf{v}^T(k) \mathbf{v}(k)]| < 1 \quad (35)$$

Note that

$$E[\mathbf{v}^T(k) \mathbf{v}(k)] = (2L+1)\sigma_v^2 \quad (36)$$

where σ_v^2 is the variance of $v(k)$, giving the bound on the step-size in the form

$$0 < \mu < \frac{2}{(2L+1)\sigma_v^2} \quad (37)$$

References

- [1] M. M. Begovic, P. M. Djuric, S. Dunlap, and A. G. Phadke, "Frequency tracking in power networks in the presence of harmonics," IEEE Trans. Power Del., vol. 8, no. 2, pp. 480-486, 1993.
- [2] S. Han, S. Han, and K. Sezaki, "Development of an optimal vehicle-to-grid aggregator for frequency regulation," IEEE Trans. Smart Grid, vol. 1, no. 1, pp. 65-72, 2010.
- [3] M. Kayikci and J. V. Milanovic, "Dynamic contribution of DFIG-based wind plants to system frequency disturbances," IEEE Trans. Power Syst., vol. 24, no. 2, pp. 859-867, 2009.
- [4] M. S. Sachdev and M. M. Giray, "A least error squares technique for determining power system frequency," IEEE Trans. Power App. Syst., vol. PAS-104, no. 5, pp. 1025-1038, 1983.
- [5] G. P. Hancke, "The optimal frequency estimation of a noisy sinusoidal signal," IEEE Trans. Instrum. Meas., vol. 39, no. 6, pp. 843-846, 1990.
- [6] T. Radil, P. M. Ramos, and A. C. Serra, "New spectrum leakage correction algorithm for frequency estimation of power system signals," IEEE Trans. Instrum. Meas., vol. 58, no. 5, pp. 1670-1679, 2009.
- [7] D. W. P. Thomas and M. S. Woolfson, "Evaluation of frequency tracking methods," IEEE Trans. Power Del., vol. 16, no. 3, pp. 367-371, 2001.
- [8] V. Kaura and V. Blasko, "Operation of a phase locked loop system under distorted utility conditions," IEEE Trans. Ind. Appl., vol. 33, no. 1, pp. 58-63, 1997.
- [9] H. Karimi, M. Karimi-Ghartemani, and M. R. Iravani, "Estimation of frequency and its rate of change for applications in power systems," IEEE Trans. Power Del., vol. 19, no. 2, pp. 472-480, 2004.

2004.

- [10] V. V. Terzija, "Improved recursive Newton-type algorithm for frequency and spectra estimation in power systems," *IEEE Trans. Instrum. Meas.*, vol. 52, no. 5, pp. 1654-1659, 2003.
- [11] Y. Xia and D. P. Mandic, "Widely linear adaptive frequency estimation of unbalanced three-phase power system," *IEEE Trans. Instrum. Meas.*, vol. 61, no. 1, pp. 74-83, 2012.
- [12] Y. Xia, S. C. Douglas, and D. P. Mandic, "Adaptive frequency estimation in smart grid applications: Exploiting noncircularity and widely linear adaptive estimators," *IEEE Signal Process. Mag.*, vol. 29, no. 5, pp. 44-54, Oct. 2012.
- [13] P. K. Dash, R. K. Jena, G. Panda, and A. Routray, "An extended complex Kalman filter for frequency measurement of distorted signals," *IEEE Trans. Instrum. Meas.*, vol. 49, no. 4, pp. 746-753, 2000.
- [14] A. Routray, A. K. Pradhan, and K. P. Rao, "A novel Kalman filter for frequency estimation of distorted signals in power systems," *IEEE Trans. Instrum. Meas.*, vol. 51, no. 3, pp. 469-479, 2002.
- [15] J. Wu, J. Long, and J. Wang, "High accuracy, wide range frequency estimation methods for power system signals under nonsinusoidal conditions," *IEEE Trans. Power Del.*, vol. 20, no. 1, pp. 366-374, 2005.
- [16] A. López, J. C. Montano, M. Castilla, J. Guitiérrez, M. D. Borrás, and J. C. Bravo, "Power system frequency measurement under nonstationary situations," *IEEE Trans. Power Del.*, vol. 23, no. 2, pp. 562-567, 2008.
- [17] H. Seyedi and M. Sanaye-Pasand, "A new time-domain based power system frequency estimation algorithm," *Euro. Trans. Electr. Power*, pp. 1-16, 2011.
- [18] A. Abdollahi and F. Matinfar, "Frequency estimation: A least-squares new approach," *IEEE Trans. Power Del.*, vol. 26, no. 2, pp. 790-798, 2011.
- [19] A. H. Sayed, *Fundamentals of Adaptive Filtering*, New York: Wiley, 2003.
- [20] A. K. Pradhan, A. Routray, and A. Basak, "Power system frequency estimation using least mean square technique," *IEEE Trans. Power Del.*, vol. 20, no. 3, pp. 761-766, 2005.
- [21] T. K. Rawat and H. Parthasarathy, "A continuous-time least mean-phase adaptive filter for power frequency estimation," *Electr. Power and Energy Syst.*, vol. 31, pp. 111-115, 2009.
- [22] V. Eckhardt, P. Hippe, and G. Hosemann, "Dynamic measuring of frequency and frequency oscillations in multiphase power systems," *IEEE Trans. Power Del.*, vol. 4, no. 1, pp. 95-102, 1989.
- [23] D. P. Mandic and S. L. Goh, *Complex Valued Nonlinear Adaptive Filters: Noncircularity, Widely Linear and Neural Models*, John Wiley & Sons, 2009.